TEST INFORMATION

DATE : 15.04.2015 **PART TEST-01 (PT-01)**

Syllabus : Function & Inverse Trigonometric Function, Limits, Continuity & Derivability, Quadratic Equation

REVISION DPP OF LIMITS, CONTINUITY & DERIVABILITY AND QUADRATIC EQUATION

Total Marks : 154	**Max. Time : 112 min.**	
Single choice Objective (no negative marking) Q. 1 to 15	**(3 marks 3 min.)**	**[45, 45]**
Multiple choice objective (no negative marking) Q. 16 to 32	**(5 marks, 3 min.)**	**[85, 51]**
Comprehension (no negative marking) Q.33 to 40	**(3 marks 2 min.)**	**[24, 16]**

1. Let f be a continuous real function such that f(11) = 10 and for all x, f(x) f(f(x)) = 1 then f(9) =

(A) 9 (B) $\frac{1}{9}$ (C) $\frac{10}{9}$ (D) $\frac{9}{10}$

2. The value of $\lim_{n\to\infty}\left(\frac{\sin\left\{\frac{2}{n}\right\}}{\left[2n\tan\frac{1}{n}\right]\left(\tan\frac{1}{n}\right)}+\frac{1}{n^2+\cos n}\right)^{n^2}$ where [.] = GIF and {.} = FPF, is

(A) 1 (B) 2 (C) 3 (D) 0

3. If f(x) = $\frac{1}{3}\left(f(x+1)+\frac{5}{f(x+2)}\right)$, f(x) > 0 $\forall$ x $\in$ R and $\lim_{x\to\infty} f(x)$ exist finitely then $\lim_{x\to\infty} f(x)$ =

(A) 0 (B) $\sqrt{\frac{2}{5}}$ (C) $\sqrt{\frac{5}{2}}$ (D) ∞

4. Let f(x) be a function which is differentiable everywhere any number of times and $f(2x^2-1) = 2x^3 f(x)$ $\forall$ x $\in$ R then $f^{2010}(0)$ is equal to ($f^n(x)$ is n^{th} derivative of f(x))

(A) –1 (B) 1
(C) 0 (D) Data provided is insufficient

5. If f(x) = $\begin{cases} \frac{xg(x)}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ and g(0) = 0 = g'(0) then f'(0) is equal to

(A) 1 (B) 0 (C) –1 (D) does not exist

6. If $y = x + e^x$ then $\left(\frac{d^2x}{dy^2}\right)_{x=\ell n2}$ is

(A) $-\frac{1}{9}$ (B) $\frac{-2}{27}$ (C) $\frac{2}{27}$ (D) $\frac{1}{9}$

7. If real numbers a and b satisfy equation |a – 1| + |b – 1| = |a| + |b| = |a + 1| + |b + 1| then minimum value of |a – b| is

(A) 3 (B) 0 (C) 1 (D) 2

8. The number of solutions of equation $|x - x^2 - 1| = |2x - 4 - x^2|$ is

(A) 1 (B) 2 (C) 4 (D) 0

9. The least positive integral solution of $x^2 - 4x > \cot^{-1}x$ is

(A) 1 (B) 2 (C) 5 (D) 4

10. The value of $\lim_{x\to\frac{\sqrt{3}}{2}} \dfrac{\frac{\pi}{3} - \sin^{-1}\left(2x\sqrt{1-x^2}\right)}{x - \frac{\sqrt{3}}{2}}$ is

(A) 1 (B) 2 (C) 3 (D) 4

11. $\left(\lim_{n\to\infty} \sum_{r=1}^{n} \left[\frac{1}{2^r}\right]\right)^m$, (where [.] denotes GIF) is equal to

(A) 0 for $m \in R$ (B) 0 for $m > 0$

(C) ∞ for $m < 0$ (D) 1 for $m = 0$

12. If $f(x) = \begin{cases} \dfrac{(e^x - 1)|x\sin x|}{x^n}, & x > 0 \\ 0 \quad , & x = 0 \end{cases}$ is derivable at x = 0 then interval of possible values of 'n' is

(A) $[1, \infty)$ (B) $(-\infty, 1]$ (C) $(-\infty, 2]$ (D) [1, 2]

13. If f(x) is a differentiable function satisfying $f(x + y) = f(x) f(y)\ \forall\ x, y \in R$ and $f'(0) = 2$ then f(x) =

(A) e^{2x} (B) $2e^x$ (C) $-e^{2x}$ (D) $-2e^x$

14. If $f(x) = ax^2 + bx + c$ such that $f(p) + f(q) = 0$ where $a \neq 0$; $p, q \in R$ then number of real roots of equation f(x) = 0 in interval [p, q] is

(A) Exactly one (B) at least one (C) at most one (D) data provided is insufficient

15. If $a = \dfrac{25 - x^2}{16}$ and $\log_a \dfrac{(24 - 2x - x^2)}{14} > 1$, then $x \in$

(A) (0, 1) (B) $(-3, 1) \cup (3, 4)$ (C) $(-\infty, -17) \cup (1, \infty)$ (D) (4, 5)

16. If f(x) is a polynomial of degree five with leading coefficient one such that $f(1) = 1^2$, $f(2) = 2^2$, $f(3) = 3^2$, $f(4) = 4^2$, $f(5) = 5^2$ then

(A) f(6) = 156 (B) $\lim_{x\to\infty} \dfrac{f(x)}{x^5} = 1$ (C) f(0) < 0 (D) f(0) > 0

17. If $f(x) = (1 + [x])^{\frac{1}{x - \{x\}}}$ where [x] and {x} denote the integral part and fractional part of x respectively and $a = \lim_{x\to 0^-} f(x)$, $b = \lim_{x\to 0^+} f(x)$, $c = \lim_{x\to 0} f(x)$ then

(A) only 'a' exists (B) b = 1

(C) a and c does not exist (D) None of a, b, c exists

18. If x = a satisfies equation $\tan^{-1}(x + 2) + \cot^{-1}\sqrt{4x + 20} = \lim_{x\to\frac{\pi}{2}} \dfrac{\sin(x\cos x)}{\cos(x\sin x)}$ and

$$\lim_{x\to 0} \frac{\sqrt[a]{1+x} - \frac{(1+x)^{\frac{1}{x}}}{e} - bx}{5x + kx^2 + x^3} = 0 \text{ then}$$

(A) a = 2 (B) ab = 3 (C) a = 1, b = 3 (D) $k \in R$

19. If $f(x) = \begin{cases} \dfrac{[\tan x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$, where [.] = GIF, then

(A) $\lim_{x\to 0^-} f(x) = 1$ (B) $\lim_{x\to 0^+} f(x) = 1$ (C) $\lim_{x\to 0^+} f(x) = 0$ (D) f(0) = 0

20. If f(x) = $\begin{cases} \frac{1}{x^2} - \frac{1}{x^2} & , \quad x < 0 \\ \sin^{-1}(x+b) & , \quad x \ge 0 \end{cases}$ then at x = 0, f(x) is

(A) continuous if b = 0
(B) discontinuous for any real b
(C) differentiable for b = ± 1
(D) non-differentiable for any real b

21. Consider a continuous function f:[0, ∞) → [0, ∞). If f(ab) = f(a) f(b) for all a, b in the domain of ' f ' and $\lim_{x\to\infty} f(x)$ is a non-zero finite number then

(A) f(2) = 2
(B) $\sum_{r=1}^{10} f(r) = 55$
(C) $\sum_{r=0}^{10} f(r) = 11$
(D) f'(2) = 0

22. If f(x) = $\left|\left|\sin\left(|x|-1\right)\right|-2\right|$ then

(A) f(x) is continuous at x = 2
(B) f(x) is differentiable at x = 2
(C) f'(2) = cos1
(D) f(x) is non-differentiable at x = 0

23. A quadratic equation f(x) = $ax^2 + bx + c = 0$ has positive distinct roots reciprocal of each other. Which one is correct?

(A) f'(1) = 0
(B) af'(1) < 0
(C) c ≠ 0
(D) abc ≠ 0

24. If the roots of the equation $(\lambda - 2)(x^2 + x + 1)^2 - (\lambda + 2)(x^4 + x^2 + 1) = 0$ are real and equal for $\lambda = \lambda_1, \lambda_2$ then

(A) $\lambda_1 + \lambda_2 = 0$
(B) $|\lambda_1 - \lambda_2| = 6$
(C) $\lambda_1^2 + \lambda_2^2 = 32$
(D) all of these

25. For all λ ∈ R, the quadratic equation $ax^2 + (b - \lambda)x + (a - b - \lambda) = 0$ has real roots. Then which of the following may be true?

(A) a = b
(B) b < a < 0
(C) b > a > 0
(D) a > b > 0

26. Which of the following is a subset of solution set of inequality $|x^2 - 2x| + |x - 4| > |x^2 - 3x + 4|$

(A) (0, 2)
(B) (4, ∞)
(C) (0, 3)
(D) [4, 5)

27. If p = $\lim_{n\to\infty} n^{-n^2} \{(n + 2^{\circ})(n + 2^{-1})(n + 2^{-2}) \ldots (n + 2^{-n+1})\}^n$ then which of the following is true?

(A) p is irrational
(B) integer nearest to p is 7
(C) p > $\lim_{x\to\infty}\left(\frac{1+x}{x}\right)^{\ell n x - x}$
(D) all of these

28. If f(x) = [4x] + {3x} where [.] denotes GIF and {.} denotes FPF then for x∈[0, 5]

(A) number of points of discontinuity of f(x) are 25
(B) f'(0) = 0
(C) f'(x) = 3 wherever defined
(D) f(x) < 20

29. The equation 2log(x + 3) = log (αx) has only one solution if

(A) α = 12
(B) α ∈(–∞, 0)
(C) α∈(0, ∞)
(D) α ∈(0,12) ∪ {24}

30. If $x^2 + 2ax + a < 0 \ \forall \ x \in [1, 2]$ then

(A) a∈ (0, 1)
(B) a > 1
(C) a < $-\frac{4}{5}$
(D) $x^2 + 2ax + a = 0$ for some x > 2

31. $\lim_{x\to\infty}\left(\frac{ax+3}{bx+2}\right)^{bx}$ is equal to (a and b are positive)

(A) 0 if a < b
(B) e if a = b
(C) ∞ if a > b
(D) 1 if a > b

32. If f(x) = $\lim_{x\to\infty} x \sin b \ \ell n\left(\frac{\sin a + \frac{1}{x}}{\sin a}\right)$ where a, b ∈ $\left(0, \frac{\pi}{2}\right)$, then f(x) can take value(s)

(A) 0
(B) 1
(C) –2
(D) 5

Comprehension # 1 (Q. 33 to 35)

If f(x) is continuous at x = 0 then xf(x) is differentiable at x = 0. By changing origin we can say that if f(x – a) is continuous at x = a then (x – a) f(x – a) is differentiable at x = a

33. The largest set over which $\frac{x\sin|x|}{1-|x|^2}$ is differentiable is

(A) R – {0, 1, –1} (B) R (C) R – {1, –1} (D) None of these

34. The number of points where $f(x) = (x-3)\,|x^2 - 7x + 12| + \cos|x-3|$ is not differentiable is

(A) 1 (B) 2 (C) 3 (D) infinite

35. Let f(x) = |x|, g(x) = sinx and h(x) = g(x) f(g(x)) then

(A) h(x) is continuous but not differentiable at x = 0
(B) h(x) is continuous and differentiable everywhere
(C) h(x) is continuous everywhere and differentiable only at x = 0
(D) all of these

Comprehension # 2 (Q. No. 36 to 37)

If left hand derivative and right hand derivative of a function is same and finite then the function is continuous as well as differentiable. If left hand and right hand derivatives are different but finite then the function is continuous but not differentiable. If one of the derivatives is infinite then the function may be continuous but not differentiable.

36. If $f(x) = \int_0^x [t]\,dt$ then

(A) f(x) is continuous and differentiable at $x \in N$ (B) f(x) is continuous but not differentiable at $x \in N$
(C) f(x) is discontinuous at $x \in N$ (D) f(x) is continuous and differentiable at $x \in Q$

37. $f(x) = \frac{x}{1+2^{1/x}}$ is

(A) continuous at all points (B) differentiable at all points
(C) continuous for $x \in R - \{0\}$ (D) non-differentiable at x = 0, 1

Comprehension # 3 (Q. No. 38 to 40)

$\phi : R \to R$ is a continuous function satisfying relation $\phi(x) - 2\phi\left(\frac{x}{2}\right) + \phi\left(\frac{x}{4}\right) = x^2$ and $\phi(0) = 1$.

38. The graph of y = ϕ(x) is a

(A) parabola (B) ellipse (C) hyperbola (D) circle

39. One of the vertices of the conic is

(A) (1, 0) (B) (0, 1) (C) (1, 1) (D) $\left(\frac{1}{2},\frac{1}{2}\right)$

40. Length of latus rectum of conic is

(A) $\frac{9}{16}$ (B) $\frac{16}{25}$ (C) $\frac{16}{9}$ (D) $\frac{25}{16}$

ANSWER KEY

DPP # 1

1.	(B)	**2.**	(D)	**3.**	(A)	**4.**	(A)	**5.**	(B)
6.	(C)	**7.**	(A)	**8.**	(D)	**9.**	(B)	**10.**	(C)
11.	(AB)	**12.**	(BD)	**13.**	(ABC)	**14.**	(ACD)		
15.	(ABCD)	**16.**	(ABCD)	**17.**	(ABC)	**18.**	(ABCD)	**19.**	(ABC)
20.	(ACD)	**21.**	(AB)	**22.**	(BC)	**23.**	(BCD)	**24.**	(ABD)
25.	(ABC)	**26.**	(AB)	**27.**	(BCD)	**28.**	(ABC)	**29.**	(AC)
30.	(ABD)	**31.**	(AB)	**32.**	(ACD)	**33.**	(B)	**34.**	(A)
35.	(D)	**36.**	(C)	**37.**	(B)				
38.	(A) → (p), (B) → (p), (C) → (p, q, s), (D) → (s)					**39.**	5	**40.**	4